CONFIDENTIAL*

954/2 STPM 2006

MATHEMATICS T

PAPER 2

Three Hours

PEPERIKSAAN PERCUBAAN STPM

DAERAH MUAR

2006

Instructions to candidates

*Answer **all** questions.*
All necessary working should be shown clearly.
Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

This question paper consists of 4 printed pages

1. PA and PB are tangents to a circle from an external point P. C is a point on the minor arc of AB. AC meets BP at D and BC meets AP at E.

   Show that $\angle PAC + \angle PBC + \angle DCE = 180^0$ [5 marks]

2. In a Δ ABC , D is the midpoint of BC & E is the midpoint of CA. The line AD ,BE of the intersect at X. Prove that

   a) the area of the Δ DEC is equal to half the area of the Δ ABD. [4 marks]

   b) the Δs AXE ,BXD are equal in area. [3 marks]

3. A ,B ,C and D are points with position vectors i + 2 j , 3i ,4i +5j and 7i + 4j respectively.

   a) show that AB is perpendicular to AC. [3 marks]

   b) show that AC and BD are parallel. [2 marks]

   c) find the angle between BC and BD [3 marks]

4. Ship A which is travelling at a speed of 15 km/hr in the direction S 45^0 E picks up ship B on its radar screen. At that instant ,ship B which is travelling at a speed of 20 km/hr in the direction S75^0E is 50 km in the direction S40^0W of ship A. If both ships maintain these velocities , find the velocity of A relative to B and the shortest distance between the ships in the ensuing motion.

   [8 marks]

5. Prove the identity $\dfrac{1}{\sin 2x}+\dfrac{1}{\tan 2x}=\cot x$ [3 marks]

   Hence

   a) Show that $\cot 15^0 = 2+\sqrt{3}$ [3 marks]

   b) solve the equation $\dfrac{1}{\sin 3\theta}+\dfrac{1}{\tan 3\theta}=2$, giving your answers in the interval $0 < \theta < 180^0$,to the nearest 0.1^0. [3 marks]

6. A territory will support a maximum population of P_o . Let the ratio of population P to the maximum population be $\dfrac{P}{P_o} = p$

   The rate of change of this ratio p is proportional to the product of p and the difference between p and 1.

   Write down a differential equation in p [2 marks]

Show that the growth of population is greatest when P is $\frac{1}{2}P_o$ [3 marks]

The population is initially $\frac{1}{4}P_o$, and reaches $\frac{1}{2}P_o$ after 20 years. Find the time for the population to reach $\frac{7}{8}P_o$? [7 marks]

Give the value of the maximum growth of population correct to 3 decimal places. [2 marks]

7. Fifteen athletes compete in the triple-jump event at a 'Majlis Sukan Sekolah-Sekolah Malaysia Daerah Muar' meeting. The distances (D m) that they jump are normally distributed according to $D = N(17.1, 0.04)$.
In the first half of the competition, the athletes have to jump a distance of 17.15 m to qualify for the final section. How many athletes would you expect, the nearest whole number, to achieve this? [5 marks]

8. Three men, A, B and C agree to meet at the theatre. The man A cannot remember whether they agreed to meet at the Cathay or the Rex and tosses a coin to decide which theatre to go to. The man B also tosses a coin to decide between the Rex and the Victory. The man C tosses a coin to decide whether to go to the Cathay or not and in this latter case he tosses again to decide between the Rex and the Victory. Find the probability that
(a) A, B and C all meet [2 marks]
(b) A, B and C all go to different places, [2 marks]
(c) at least two meet. [2 marks]

9. The number of people entering a railway station between the hours of 7.30 and 9.30 a.m. on a weekday is a Poisson variate and averages two people per minute. Find the probability that
(a) five people enter in one minute, [2 marks]
(b) less than four people enter in a two-minute period. [3 marks]

10. The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} k(2-x), 0 \le x \le 2 \\ 0, otherwise \end{cases}$$

where k is a constant.

(a) Find the value of k. [2 *marks*]

(b) Find the cumulative distribution function of X. [4 *marks*]

The continuous random variable Y is given by $Y = 1 - \frac{1}{2}X$. Show that $P(Y < y) = y^2$, where $0 \le y \le 1$. [4 *marks*]

11 A discrete random variable X has the probability distribution given in the following table.

x	2	3	4	5
$P(X = x)$	P	$\frac{1}{5}$	$\frac{3}{10}$	q

(a) Given that $E(X) = 4$, find p and q. [6 *marks*]

(b) Show that $Var(X) = 1$. [3 *marks*]

(c) Find $E(|x - 4|)$. [2 *marks*]

12 The following data show the masses, in kg, of fish caught by 25 fishermen on a particular day.

10	8	7	4	4	18	9	14	17	11	5	4	5
7	10	12	14	10	9	29	7	6	5	20	15	

(a) Find the mean and standard deviation. [5 *marks*]

(b) Find the percentage of the fishermen with the mass of fish within one standard deviation from the mean. [3 *marks*]

(c) Construct a frequency distribution by grouping the masses of fish according to class intervals 1 − 5, 6 − 10, 11 − 15, 16 − 20 and 21 − 30. Hence, plot a histogram corresponding to this grouped data. [4 *marks*]